33.57. Model: Assume the magnetic field is constant throughout the atmosphere.
Visualize: The energy density depends on the field strength in a particular region of space. The total energy is the energy density times the volume.
Solve: **(a)** The atmosphere is a spherical shell between the surface of the earth at R_e and the top of the atmosphere at $R_e + h$ where h is the thickness of the atmosphere. We have

$$u_B = \frac{1}{2\mu_0}B^2 = \frac{\left(50\times10^{-6}\ \text{T}\right)^2}{2\left(4\pi\times10^{-7}\ \text{T m / A}\right)} = 9.95\times10^{-4}\ \text{J / m}^3$$

$$V_{\text{atm}} = \tfrac{4}{3}\pi\left[\left(R_e + h\right)^3 - R_e^{\,3}\right] = \tfrac{4}{3}\pi\left[\left(6.37\times10^6\ \text{m} + 20\times10^3\ \text{m}\right)^3 - \left(6.37\times10^6\ \text{m}\right)^3\right] = 1.02\times10^{19}\ \text{m}^3$$

$$\Rightarrow U_{\text{tot}} = u_B V_{\text{atm}} = \left(9.95\times10^{-4}\ \text{J / m}^3\right)\left(1.02\times10^{19}\ \text{m}^3\right) = 1.0\times10^{16}\ \text{J}$$

(b) The ratio is

$$\frac{U_{\text{mag}}}{U_{\text{tot}}} = \frac{1.0\times10^{16}\ \text{J}}{4.0\times10^{18}\ \text{J}} = 0.25\times10^{-2} = 0.25\ \%$$

Assess: Not enough to do much good.